4.3

$D = 10^{-5}\ cm^2\ s^{-1}$

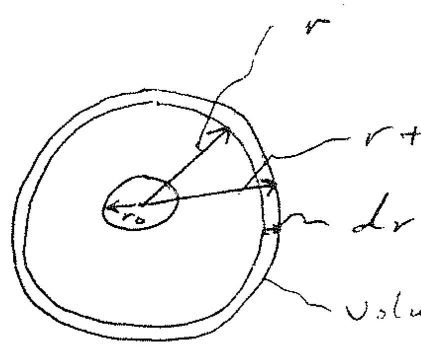
Min distance btw WE and cell wall = $5 \cdot \ell$

$$= 5 \cdot \sqrt{2Dt} = 5 \cdot \left(2 \cdot 10^{-5} \frac{cm^2}{s} \cdot 10^2\ s\right)^{1/2}$$

$$= 0.22\ cm \sim 220\ \mu m$$

4.5

$O + ne \leftrightarrow R$



volume of "shell" is $4\pi r^2 dr$

FFL for the sphere is

$$J_O(r,t) = -\frac{D_O\,\partial C_O(r,t)}{\partial r}$$

The volume element of thickness dr contained between (r) and (r+dr) must be inspected.

The change in # of moles of O, N_O, within the above-mentioned volume element over time interval dt is

moles diffusing in across boundary(r) − # moles diffusing out across boundary (dr)

$$= \overbrace{4\pi r^2}^{\text{Area at } r} \cdot \overbrace{J_O(r,t)}^{\text{Flux}}\, \overbrace{dt}^{\text{time}} - \overbrace{4\pi(r+dr)^2}^{\text{Area at } r+dr} * \underbrace{J_O(r+dr,t)}_{\text{flux at } r+dr}\, dt$$

4.5 (cont'd)

This is equal to dN_0

$$dN_0 = 4\pi r^2 \cdot J_0(r,t)\,dt - 4\pi(r+dr)^2 J_0(r+dr,t)\,dt$$

Okay, from your text in discussion on FSC:

$$J_0(r+dr, t) = J_0(r,t) + \frac{\partial J_0(r,t)}{\partial r}\,dr$$

$$dN_0 = 4\pi r^2 J_0(r,t)\,dt - 4\pi(r+dr)^2\left[J_0(r,t) + \frac{\partial J_0(r,t)}{\partial r}\,dr\right]_{dt}$$

$$dN_0 = 4\pi r^2 J_0(r,t)\,dt - 4\pi(r+dr)^2 J_0(r,t)\,dt - 4\pi(r+dr)^2 \frac{\partial J_0(r,t)}{\partial r}\,dr\,dt$$

$$dN_0 = 4\pi\,dt\left[r^2 J_0(r,t) - (r+dr)^2 J_0(r,t) - (r+dr)^2 \frac{\partial J_0(r,t)}{\partial r}\,dr\right]$$

$$dN_0 = 4\pi\,dt\left[J_0(r,t)\left(r^2 - (r+dr)^2\right) - (r+dr)^2 \frac{\partial J_0(r,t)}{\partial r}\,dr\right]$$

$$dC_0(r,t) = \frac{dN_0}{V} = \frac{dN_0}{4\pi r^2\,dr}$$

$$dC_0(r,t) = dt\left[J_0(r,t)\,\frac{\left(r^2 - (r+dr)^2\right)}{r^2\,dr} - \frac{(r+dr)^2}{r^2}\,\frac{\partial J_0(r,t)}{\partial r}\right]$$

$$\frac{dC_0(r,t)}{dt} = \left[J_0(r,t)\cdot\left(\frac{\cancel{r^2} - r^2 - 2r\,dr - (dr)^2}{r^2\,dr}\right) - \frac{\partial J_0(r,t)}{\partial r}\left(\frac{r^2 + 2r\,dr + (dr)^2}{r^2}\right)\right]$$

4.5 (cont'd)

$$\frac{d\,C_o(r,t)}{dt} = \left[-J_o(r,t)\left(\frac{2}{r} + \frac{dr}{r^2}\right) - \frac{\partial J_o(r,t)}{\partial r}\left(1 + \frac{2dr}{r} + \frac{(dr)^2}{r^2}\right)\right]$$

$$\frac{d\,C_o(r,t)}{dt} = \frac{\partial C_o(r,t)}{\partial t}$$

and $dr \approx 0$

$$\frac{\partial C_o(r,t)}{\partial t} = \left[-J_o(r,t)\left(\frac{2}{r}\right) - \frac{\partial J_o(r,t)}{\partial r}(1)\right]$$

$$\frac{\partial C_o(r,t)}{\partial t} = -\frac{J_o(r,t)\cdot 2}{r} - \frac{\partial J_o(r,t)}{\partial r}$$

But, FFL is $J_o(r,t) = -D_o \dfrac{\partial C_o(r,t)}{\partial r}$

$$\frac{\partial C_o(r,t)}{\partial t} = D_o\left[\frac{2}{r}\cdot\frac{\partial C_o(r,t)}{\partial r} + \frac{\partial^2 C_o(r,t)}{\partial r^2}\right]$$

Q.E.D. ✓